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Tactful Management

AN INVENTORY MODEL FOR DETERIORATING ITEMS TAKING DEMAND RATE DEPENDENT ON BOTH SELLING PRICE AND INVENTORY LEVEL

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1.1 General

The present paper deals with a deteriorating inventory model considering price and inventory level dependent demand with variable rat deterioration. The model is developed under more realistic assumptions considering shortages and instantaneous delivery.Expressions are obtained for order quantity and initial inventory after fulfilling backorder and minimum cost criterion is used to solve the model substituting appropriate value of the parameter the model reduces to known result.

Keywords: variable rat deterioration , marketing policies.

1.2 Introduction

The most important assumption in the classical inventory models found in the existing literature is that the life time of an item is infinite while it is in storage. But the effect of deterioration plays an important role in the storage of some commonly used physical goods like fruits, vegetables etc. In these cases, a certain fraction of these goods are either damaged or decayed and are not in a condition to satisfy the future demand of customers as fresh units. Deterioration in these units is continuous in time and is normally proportional to on-hand inventory. Recently, a good number of works have been done by some authors for controlling inventories in which a constant or variable part of the on-hand inventory gets deteriorated per unit of time.

In the competitive market situation, customers are influenced by the marketing policies such as the price variations of an item and the attractive display of units in the show-room at the business places. By modern lighting and electronic arrangement, glamorous display of units in large numbers has a motivational effect on the people and attracts the customers to buy more. In the last few years, many researchers have focused on the analysis of inventory system which describes the demand rate dependent on the displayed inventory level.

Again, the selling price is one of the decisive factors in selecting an item for use. It is well known that lesser the selling price of an item increases the demand of that item whereas higher selling prices has the reverse effect. Hence, it can be concluded that the demand of an item is a function of both selling price and the instantaneous stock-level of that item. However, very few OR researchers and practitioners studied the effects of price variations. Incorporating the effects of selling price on demand.

Basu etal (6) developed an ordering policy for deteriorating items with two component demand and price breaks allowing shortages. Dave (18) developed an order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replenishment. Goel and Aggarwal (27), Gor and Shah (28), Pal and Mandal (61), Shah and Jaiswal (71) developed order level inventory model by taking different by taking different demand and deterioration rates.

1.3 Assumptions and Notations:-

The model is developed under the following assumptions and notations.

(i) The demand rate is assumed as $D(p,t) = a\text{-}bp+\gamma I\ (t)$ is demand rate at price ‘p’ and time ‘t’. I(t) is the on hand inventory at time ‘t’ a, b and γ are positive constant and ‘p’ is the selling price per unit.

(ii) Lead time is zero.

(iii) T is the fixed length of each production cycle.

(iv) C_1 is the inventory holding cost per unit per unit time.

(v) C_2 is storage cost per unit per unit time.

(vi) C_3 is the cost of each deteriorated unit.

(vii) Shortages are allowed and fully backlogged.

(viii) A variable fraction $\theta(t)$ of the on hand inventory deteriorated per unit time. $\theta(t)$ is assumed in the form $\theta(t) = \alpha\beta t^{\beta-1}$ which is a two parameter weibull distribution function α is the scale parameter, $\alpha>0$. B>0, and ‘t’ is time of deterioration, ‘t’>0.

(ix) The replenishment rate is infinite and replenishment size is constant.

1.4 The Mathematical Model:-

Let Q be the total amount of inventory produced or purchased at the beginning of each period and after fulfilling back orders let us assume, we get an amount S(S>0) an initial inventory. Inventory level gradually diminishes during time period $(0, t_1)$, $t_1 < T$ due to the reasons of market demand and deterioration of items and ultimately falls to zero at time $t = t_1$. Shortages occur during time period (t_1, T) which are fully backlogged. Let I(t) be the on hand inventory at any time t. The differential equations which the on hand inventory I(t) must satisfy in two parts of the cycle time T are the following.

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -\left(a - bp + \gamma I(t)\right) \qquad 0 \le t \le t_1 \quad (1.1.1)$$

$$\frac{dI(t)}{dt} = -[a - bp] \qquad t_1 \le t \le T \quad (1.1.2)$$

Solution of differential equation (1.4.1) is given by

$$\frac{dI(t)}{dt} + \left(\alpha\beta t^{\beta-1} + \gamma\right) I(t) = -(a - bp)$$

$$I(t).e^{\int(\alpha\beta t^{\beta-1}+\gamma)dt} = -(a - bp)\int e^{\int(\alpha\beta t^{\beta-1}+\gamma)dt} dt + k_1$$

$$I(t).e^{\alpha t^{\beta}+\gamma t} = -(a-bp)\int e^{\alpha t^{\beta+\gamma t}}dt + k_1$$

$$= -(a-bp)\left[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\gamma t^2}{2}\right] + k_1$$

$$I(t) = -(a-bp)\left[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\gamma t^2}{2}\right]\exp.\left(-(\alpha t^{\beta} + \gamma t)\right)$$

$$+k_1 \exp.\left(-(\alpha t^{\beta} + \gamma t)\right)$$

Using initial condition

At $t = 0$, $I(0) = S$

$S = 0.e^{0}.+k_1 \Rightarrow k_1 = S$

$$\Rightarrow \quad I(t) = -(a-bp)\left[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\lambda t^2}{2}\right]\exp.(-\alpha t^{\beta} - \gamma t)$$

$$+ S\exp.(-\alpha t^{\beta} - \gamma t) \tag{1.1.3}$$

Solution of (1.1.2) is given by

$$\frac{dI(t)}{dt} = -(a-bp)$$

$$dI(t) = -(a-bp)dt$$

Integrating both sides

$$I(t) = -(a-bp)t + k_2 \tag{1.1.4}$$

Using Initial Condition at $t = t_1$

$$I(t_1)1 = 0$$

$$I(t_1)1 = -(a-bp)t_1 + k_2$$

$$0 = -(a-bp)t_1 + k_2$$

$$+(a-bp)t_1 = k_2$$

Substituting his value of k_2 in (4) we get

$$I(t) = (a - bp)(t_1 - t) \tag{1.1.5}$$

Since $I(t_1) = 0$ then from (3) we have

$$(a - bp)\left[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta + 1} + \frac{\gamma t_1^{2}}{2}\right] = S \tag{1.1.6}$$

Then from (1.1.3) we have

$$I(t) = -(a - bp)\left[t + \frac{\alpha \beta t^{\beta+1}}{\beta + 1} + \frac{\gamma t^{2}}{2}\right]$$

$$+(a - bp)\left[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta + 1} + \frac{\gamma t_1^{2}}{2} - \alpha t_1^{\beta} - \gamma t_1 t\right]$$

$$0 \le t \le t_1 \tag{1.1.7}$$

Total amount of deteriorated units are given by

$$= S - \int_0^{t_1} (a - bp + \gamma I(t))dt$$

$$= S - \int_0^{t_1} (a - bp)dt - \gamma \int_0^{t_1} I(t)dt$$

$$= S - (a - bp)t_1 - \gamma\left[-(a - bp)\left(\frac{t^2}{2} - \frac{\alpha \beta t^{\beta+2}}{(\beta + 1)(\beta + 2)} - \frac{\gamma t^3}{6}\right)\right.$$

$$\left. +(a - bp)\left(t_1 t + \frac{\alpha t_1^{\beta+1}}{\beta + 1}t + \frac{\gamma t_1^{2}}{2}t - \frac{\alpha t_1 t^{\beta+1}}{\beta + 1} - \frac{\gamma t_1 t^2}{2}\right)\right]_0^{t_1}$$

$$= S - (a - bp)t_1 - \gamma\left[-(a - bp)\left(\frac{t_1^2}{2} - \frac{\alpha \beta t_1^{\beta+2}}{(\beta + 1)(\beta + 2)} - \frac{\gamma t_1^3}{6}\right)\right.$$

$$\left. +(a - bp)\left(t_1^2 + \frac{\alpha t_1^{\beta+2}}{\beta + 2}t + \frac{\gamma t_1^{3}}{2}t - \frac{\alpha t_1 t^{\beta+2}}{\beta + 2} - \frac{\gamma t_1 t^3}{2}\right)\right]$$

Substituting value of S in the above equation from equation (1.1.6)

$$= \quad (a - bp)\left(t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\gamma t_1^2}{2}\right) - (a - bp)t_1$$

$$+\gamma(a - bp)\left(\frac{t_1^2}{2} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\gamma t_1^3}{6}\right) - \gamma(a - bp)t_1^2$$

$$= \quad (a - bp)\left(t_1 + \frac{\alpha t_1^{\beta+1}}{(\beta+1)} + \frac{\gamma t_1^2}{2}\right) - (a - bp)t_1$$

$$+\gamma(a - bp)\left(-\frac{1}{2}t_1^2 - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\gamma t_1^3}{6}\right)$$

$$= \quad (a - bp)\left(\frac{\alpha t_1^{\beta+1}}{(\beta+1)} + \frac{\gamma t_1^2}{2} - \frac{\gamma t_1^2}{2} - \frac{\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right)$$

$$= \quad (a - bp)\left(\frac{\alpha t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right) \qquad (1.1.7)$$

= 9609108888 (Gupta ji)

Neglecting γ^2 term

Unit cost over the period (0, T) is given by

$$= \quad C(a - bp)\left(\frac{\alpha t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right) \qquad (1.1.8)$$

Inventory holding cost over the period (0, T) is given by

$$C_1\int_0^{t_1} I(t)\,dt$$

$$= \quad C_1(a - bp)\left[\frac{t_1^2}{2} + \frac{\gamma t_1^3}{6} + \frac{\alpha\gamma t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{t_1^{\beta+3}}{2(\beta+1)}\right.$$

$$\left. - \frac{\alpha\gamma t_1^{\beta+2}}{2(\beta+1)} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] \qquad (1.1.9)$$

Shortage case is given by

$$= \qquad C_2 \int_{t_1}^{T} [-I(t)\, dt$$

$$= \qquad C_2 (a - bp) \frac{(T - t_1)^2}{2} \qquad (1.1.10)$$

Total average cost per unit time is given by C(S, t_1)

= Average unit cost + Average holding cost + Average shortage cost

$$C(S, t_1) = \frac{C_3(a - bp)}{T}\left[-\frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+ \frac{aC_1}{T}\int_0^{t_1} I(t)dt - \frac{eC_2}{T}\int_{t_1}^{T} I(t)dt \qquad (1.1.11)$$

Now substituting values for I(t) given by equations (1.1.6), (1.1.4) and eliminating S using equation (1.1.5) and then on integration we get

$$= \frac{C_3(a - bp)}{T}\left[\frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + C_1 \frac{(a - bp)}{T}$$

$$\left[\frac{t_1^2}{2} + \frac{\gamma t_1^3}{6} + \frac{\alpha\gamma t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{t_1^{\beta+3}}{2(\beta+1)} - \frac{\alpha\gamma t_1^{\beta+2}}{2(\beta+1)} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+ \frac{C_2(a - bp)}{2T}(T - t_1)^2 \qquad (1.1.12)$$

$$C(t_1) = \frac{C_3(a - bp)}{T}\left[\frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+ \frac{C_1(a - bp)}{T}\left[\frac{t_1^2}{2} + \frac{\gamma t_1^3}{6} + \frac{\alpha\gamma t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{t_1^{\beta+3}}{2(\beta+1)}\right.$$

$$\left.- \frac{\alpha\gamma t_1^{\beta+2}}{2(\beta+1)} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \frac{C_2(a - bp)}{2T}(T - T_1)^2 \qquad (1.1.13)$$

For minimum cost, the necessary condition is

$$\frac{dc(t_1)}{dt_1} = 0 \tag{1.1.14}$$

$$\Rightarrow \quad \frac{C_3(a-bp)}{T}\left[\alpha t_1^{\beta} - \frac{\alpha\beta\gamma t_1^{\beta+1}}{(\beta+1)}\right] + \frac{C_1(a-bp)}{T}\left[t_1 + \frac{\gamma t_1^2}{2}\right.$$

$$\left. + \frac{\alpha\gamma t_1^{\beta+2}}{\beta+1} + \frac{(\beta+3)t_1^{\beta+2}}{2(\beta+1)} - \frac{\alpha\gamma(\beta+2)t_1^{\beta+1}}{2(\beta+1)} + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right]$$

$$-\frac{2C_2(a-bp)}{2T}(T-T_1) = 0 \tag{1.1.15}$$

Last term of the above equation is negative. So it has at least one positive root.

The optimum value of $t_1 = t_1^*$ is obtained by the solution of equation (1.1.15). Subtracting it in (1.1.6), the optimum value of S is

$$S^* = (a-bp)\left[t_1^* + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\gamma t_1^{*2}}{2}\right] \tag{1.1.16}$$

The optimum value for Q is

$$Q^* = (a-bp)\left[t_1^* + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\gamma t_1^{*2}}{2} + (T-t_1)\right]$$

Minimum value of Cost is $C(t_1^*)$ can be obtained from equation (1.1.13)

1.5 Special Case:-

If there is no deterioration, then $\alpha = 0$

from equation (1.1.12) we have

$$t_1^* = \frac{C_2 T}{C_1 + C_2}$$

If $\beta = 1$ Then the model reduces to constant rate of deteriorating model.

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